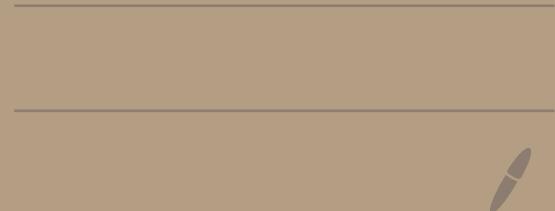


Advanced ML

& AGT

Class 2



מונט קרטזיאן

הנאה

דומם ותבונת

"הנאה הינה מושג"

Regret

לדא נחלה והזנה

$f: X \rightarrow [-1, +1]$ גורם כירען גורם:

$$E_{x \sim Q}[f(x)] - E_{x \sim P}[f(x)] \leq \|P - Q\|_1$$

12/12 f,p"p1

$$E_{x \sim Q}[f(x)] - E_{x \sim P}[f(x)] = \|P - Q\|_1$$

תגמול

$$\left| E_{x \sim Q}[f(x)] - E_{x \sim P}[f(x)] \right| = \left| \sum_x f(x)(Q(x) - P(x)) \right|$$

$$\leq \sum_x |f(x)| \cdot |Q(x) - P(x)|$$

$$\leq \sum_x |Q(x) - P(x)| = \|P - Q\|_1$$

$$f(x) = \text{sign}(Q(x) - P(x))$$

7'3כ, יلد רפוד

$$E_{x \sim Q}[f(x)] - E_{x \sim P}[f(x)] = \sum_x f(x)(Q(x) - P(x))$$

$$= \sum_x |Q(x) - P(x)| = \|P - Q\|_1$$



נוסף נסחף נסחף

תכלת

$$\|P - Q\|_1 = \sum_x |P(x) - Q(x)| = 2\|P - Q\|_{TV}$$

: גורם

$$\|P - Q\|_1 = 2 \max_A |P(A) - Q(A)|$$

תכלת:

$$A = \{x : P(x) \geq Q(x)\} \quad ?'3כ$$

$$\|P - Q\|_1 = \sum_x |P(x) - Q(x)|$$

$$= \sum_{x \in A} |P(x) - Q(x)| + \sum_{x \notin A} |Q(x) - P(x)|$$

$$= 2(P(A) - Q(A))$$

$$= 2 \max_B |P(B) - Q(B)|$$

$$\|P^m - Q^m\| = \sum_{x_m} \sum_{x_{-m}} |\alpha(x_{-m}) P_m(x_m) - \beta(x_{-m}) Q(x_m)|$$

$$\alpha(x_{-m})(P_m(x_m) - Q_m(x_m)) + Q_m(x_m)(\alpha(x_{-m}) - \beta(x_{-m}))$$

$$\leq \sum_{x_m} |P_m(x_m) - Q_m(x_m)| \underbrace{\alpha(x_{-m})}_{1} + \sum_{x_m} |\alpha(x_{-m}) - \beta(x_{-m})| \underbrace{Q(x_m)}_{1}$$

$$\leq \|P_m - Q_m\|_1 + \sum_{i=1}^{m-1} \|P_i - Q_i\|_1$$



$Q_1 \dots Q_m - 1 \quad P_1 \dots P_m$: 77387
7'387, 7'387

$$P^m = P_1 \times \dots \times P_m$$

$$Q^m = Q_1 \times \dots \times Q_m$$

$$\|P^m - Q^m\|_1 \leq \sum_{i=1}^m \|P_i - Q_i\|_1$$

m 8 7'387/7'387 : 61217

7'387 : m=1 7'387

$$P^m(x) = \prod_{i=1}^m P_i(x_i) = P_m(x_m) \alpha(x_{-m}) \quad : m \geq 1 \quad 7'387$$

$$Q^m(x) = \prod_{i=1}^m Q_i(x_i) = Q_m(x_m) \beta(x_{-m})$$

7'387/7'387 7'387

$$\|\alpha - \beta\|_1 \leq \sum_{i=1}^{m-1} \|P_i - Q_i\|_1$$

מודולו

$$P_i \sim Br\left(\frac{1}{2}\right) \quad Q_i \sim Br\left(\frac{1+\varepsilon}{2}\right) \quad \text{לפ' גודל מינימלי}$$

$$\|P_i - Q_i\|_1 = \varepsilon$$

הנחות על P^m ו- Q^m ביחס ל- f
 $\Pr_{P^m}[f=1] \geq 3/4$, $\Pr_{Q^m}[f=1] \leq 1/4$

$$\Pr_{P^m}[f=1] \leq \delta \quad \Pr_{Q^m}[f=1] \geq 1-\delta$$

$$1-2\delta \leq E_{Q^m}[f] - E_{P^m}[f] \leq \|P^m - Q^m\|_1 \leq m\varepsilon \quad \text{לפ' CN}$$

$$m \geq \frac{1-2\delta}{\varepsilon} = \Omega\left(\frac{1}{\varepsilon}\right) \quad \text{לפ'}$$

? פירוש מונט קרלו

KL divergence សេរីយោប់

$$KL(P||Q) \geq 0 \quad \text{ស្ម័គ្រាប់} \quad ①$$

$$P = Q \quad \text{និង} \quad KL(P||Q) = 0$$

និង សេរីយោប់ នូវវា : chain rule ②

$$KL(P^m || Q^m) = \sum_{i=1}^m KL(P_i || Q_i)$$

: A សេរីយោប់ សែស ③

$$\sum_{x \in A} P(x) \log \frac{P(x)}{Q(x)} \geq P(A) \log \frac{P(A)}{Q(A)}$$

A សេរីយោប់ សែស : Pinsker's inequality ④

$$2(P(A) - Q(A))^2 \leq KL(P||Q)$$

KL-Divergence

Kullback-Leibler (KL)

: ៩៧៣៤៧

$$KL(P||Q) = E_{x \sim P} \left[\log \frac{P(x)}{Q(x)} \right]$$

$$= \sum_x P(x) \log \frac{P(x)}{Q(x)}$$

សេរីយោប់ និង KL : សម្រាប់

$$KL(P||Q) \neq KL(Q||P)$$

សេរីយោប់ និង KL

$$KL(P^m || Q^m) = \sum_{i=1}^m KL(P_i || Q_i) \quad \text{Chain Rule} \quad (2)$$

בְּרִכָּה

$$x = (x_1, \dots, x_m) \quad h_i(x_i) = \log \frac{P_i(x_i)}{Q_i(x_i)} \quad \text{J'3CJ}$$

$$KL(P||Q) = \sum_x P^m(x) \log \frac{P^m(x)}{Q^m(x)}$$

$$= \sum_{\mathbf{x}} P^m(\mathbf{x}) \sum_{i=1}^m \log \frac{P_i(x_i)}{Q_i(x_i)}$$

$P_i(x_i)$

$$= \sum_{i=1}^m \sum_{\bar{x}_i} h_i(\bar{x}_i) \underbrace{\sum_{x: x_i = \bar{x}_i} p^m(x)}_{P_i(\bar{x}_i)}$$

$$= \sum_{i=1}^m \sum_{\bar{x}_i} P_i(\bar{x}_i) \log \frac{P_i(\bar{x}_i)}{Q_i(\bar{x}_i)}$$

$$KL(P_i \parallel Q_i)$$

$$KL(P||Q) \geq 0 \text{ ①}$$

$$f(y) = y \log(y) \quad 7'3\zeta J$$

הַיְלָדֶת הַיְלָדֶת אֲלֵיכֶם כִּי

$$KL(P||Q) = \sum_x P(x) \log \frac{P(x)}{Q(x)}$$

$$= \sum_x Q(x) f\left(\frac{P(x)}{Q(x)}\right) = E_Q\left[f\left(\frac{P(x)}{Q(x)}\right)\right]$$

$$\geq f\left(\sum_x Q(x) \frac{P(x)}{Q(x)}\right)$$

$$= f\left(\sum_x P(x)\right) = f(1) = 0$$

Ex $P(X) = Q_X(\rho)$ if $\rho \in \mathcal{P}_X$ then ρ is called a \mathcal{P}_X -measure.

Pinsker π-measure (4)

$$\forall A \subseteq \Omega \quad 2(P(A) - Q(A))^2 \leq KL(P||Q)$$

③ Will the two paths end

$$\sum_{x \in A} P(x) \log \frac{P(x)}{Q(x)} \geq P(A) \log \frac{P(A)}{Q(A)}$$

$$\sum_{x \in A} P(x) \log \frac{P(x)}{Q(x)} \geq (-P(A)) \log \frac{1 - P(A)}{1 - Q(A)}$$

$$b = Q(A) \quad a = P(A) : \text{new}^{\circ}$$

$$KL(P||Q) \geq a \log \frac{a}{b} + (1-a) \log \frac{1-a}{1-b}$$

$$= \int_a^b -\frac{a}{x} + \frac{1-a}{1-x} dx = \int_a^b \frac{x-a}{x(1-x)} dx$$

$$\frac{1}{4} \geq x(1-x) \quad \text{for } x \in [0, 1]$$

$$KL(P||Q) \geq \int_a^b q(x-a) dx = 2(b-a)^2 = 2(Q(A) - P(A))^2$$

$$\forall A \subseteq \Omega \quad \sum_{x \in A} P(x) \log \frac{P(x)}{Q(x)} \geq P(A) \log \frac{P(A)}{Q(A)} \quad (3)$$

$$P_A(x) = P[x|A] \quad Q_A(x) = Q[x|A] \quad \text{? 3.5}$$

$$P(x) = P(A)P_A(x) \quad : 1225$$

$$\sum_{x \in A} P(x) \log \frac{P(x)}{Q(x)} = P(A) \sum_{x \in A} P_A(x) \log \frac{P(A)}{Q(A)} \frac{P_A(x)}{Q_A(x)}$$

$$= P(A) \sum_{x \in A} P_A(x) \log \frac{P_A(x)}{Q_A(x)} + P(A) \log \frac{P(A)}{Q_A} \sum_{x \notin A} P_A(x)$$

$$KL(P_A || Q_A) \geq 0$$

$$\geq P(A) \log \frac{P(A)}{Q(A)}$$

ווד ערך גנריון

$$Q = Br(k) \quad Br\left(\frac{1+\varepsilon}{2}\right) = P \quad \text{ווד גנריון} \quad \text{je } \varepsilon' \\ \varepsilon \leq \frac{1}{2}$$

$$KL(P||Q) = \frac{1+\varepsilon}{2} \log(1+\varepsilon) + \frac{1-\varepsilon}{2} \log(1-\varepsilon)$$

$$= \underbrace{\frac{1}{2} \log(1-\varepsilon^2)}_{\text{Side}} + \underbrace{\frac{\varepsilon}{2} \log\left(\frac{1+\varepsilon}{1-\varepsilon}\right)}_{\log\left(1 + \frac{2\varepsilon}{1-\varepsilon}\right)}$$

$$\leq \frac{2\varepsilon}{1-\varepsilon} \leq 4\varepsilon$$

$$KL(P||Q) \leq 2\varepsilon^2$$

$$KL(P^m || Q^m) \leq 2m\varepsilon^2$$

L1 ווד כינר פון

$$2(1-2\varepsilon)^2 \leq KL(P^m || Q^m) \leq 2\varepsilon^2 m$$

: גדר

ווד גנריון יתפסת כב' צד

$$m \geq \frac{(1-2\varepsilon)^2}{\varepsilon^2} = \mathcal{O}\left(\frac{1}{\varepsilon^2}\right)$$

? פונקציית האפקט מוגדרת

(כ) $T \leq \frac{c k}{\varepsilon^2}$ וע"מ: גונס

א מ"מ $k/3$ מינימום נ"מ' גונס

$$\Pr[Y_T = a | I_a] < 3/4 \quad \text{ולכן} \quad \Pr[Y_T = a | I_a] < 3/4$$

$$T = \Omega(Y_T^2) \geq k/3 \quad \text{ר"ג} \quad \textcircled{R}$$

$$T = \Omega(k/\varepsilon^2) \geq k/3 \quad k \geq 24 \quad \text{ר"ג} \quad \textcircled{S}$$

הכליה 1/3

$$I_1 = (0.6, 0.5, 0.5)$$

$$I_2 = (0.5, 0.6, 0.5)$$

$$I_3 = (0.5, 0.5, 0.6)$$

Best Arm "הטובה ביותר": δ_{MN}

, T מ"מ δ_{MN} -

$y_T \in A$ $k \cdot 3/4$

$$\Pr[Y_T = a^*] : \text{ונ"מ} -$$

$$\mu(a) = E[r_t(a)] , \quad r_t(a) \text{ מ"מ} : a \text{ מ"מ} -$$

$$I = \{\mu(a) : a \in A\} \quad \text{הכליה} -$$

הכליה δ_{MN} (גונס)

$$I_j = \begin{cases} \mu(i) = \frac{1}{2} & i \neq j \\ \mu(i) = \frac{1+\varepsilon}{2} & i = j \end{cases}$$

$\forall j$

$$0.99 \leq \Pr[Y_T = j | I_j] : \text{ונ"מ}$$

$$T = \Omega\left(\frac{k}{\varepsilon^2}\right) : \text{הכליה}$$

$$2 \left(\overbrace{P_1(A) - P_2(A)}^{V_2} \right)^2 \leq KL(P_1 \| P_2) \quad \text{is true}$$

$$= \sum_{a \in \{1,2\}} \sum_{t=1}^T KL(P_1^{a,t} \| P_2^{a,t})$$

$$\leq 2T \cdot 2\varepsilon^2 = 4T\varepsilon^2$$

$$\frac{1}{8} \cdot \frac{1}{\varepsilon^2} \leq T$$

T is enough now



$k=2$ case

Now consider the case $k=2$, we have

$y_T=1$ means we pick A randomly
Now consider the pick

$$P_1(A) = Pr[y_T=1 | I_1] \geq \frac{3}{4}$$

$$P_2(A) = Pr[y_T=1 | I_2] \leq \frac{1}{4}$$

$$E_0[T_j] \leq \frac{3T}{K} \Rightarrow P_0\left[T_j \geq \frac{24T}{K}\right] \leq \frac{1}{8} \quad (3)$$

↓

$$P_0\left[T_j \leq \frac{24T}{K}\right] \geq \frac{7}{8}$$

$$(K_3 = K_1 \cap K_2) \quad \text{ר'ז סדר קבוצת מינימום} \quad (4)$$

$$P_0\left(T_j \leq \frac{24T}{K}\right) \geq \frac{7}{8} \neq P_0(y_r=j) \leq \frac{3}{K}$$

ליכוד כבש 7/28

$$I_0 = \{ \mu(a) = y_2 \} \quad f_{01} \in \mathcal{F}_0 \subset \mathcal{F}_{01}$$

$$E_0[x] = E[x|I_0]$$

$$P_0(A) = P_r[A|I_0] \quad \mu_0$$

- j נסובך יגנ'ה נסוב - T_j

j נסוב נסוב $\frac{2}{3}K$ גאנט ①

$3T/K \geq E_0[T_j]$ ר'ז סען

(k_1, 23/27)

j נסוב נסוב $2/3$ גאנט ②

$P_0(y_r=j) \leq \frac{3}{K}$ ר'ז סען

(k_2, 23/27)

$$\forall A \subseteq \Omega^* \quad |P_o^*(A) - P_j^*(A)| \leq \varepsilon \sqrt{m} \leq \frac{1}{8} \quad \text{Nedon}$$

$\Omega^* \supseteq \mathcal{I}^*$ 'def' $\mathcal{I}^* \cap \mathcal{I}_j$ $\mathcal{I}_j \in \mathcal{P}(3)$
 $\vdash \mathcal{I}^* \in \mathcal{I}$

$$A = \{y_T = j, T_j \leq m\} \neq A' = \{T_j > m\}$$

$\mathcal{I}^* \supseteq \mathcal{I}_j$ $\mathcal{I}_j \in \mathcal{I}$

$$P_j(A) \leq \frac{1}{8} + P_o(A) \leq \frac{1}{8} + P_o[y_T = j] \leq \frac{1}{8} + \frac{3}{K} \leq \frac{1}{4}$$

$$P_j(A') \leq \frac{1}{8} + P_o(A') \leq \frac{1}{4}$$

: pl. 0.8

$$P_j(y_T = j) \leq P_j^*(y_T = j, T_j \leq m) + P_j^*(T_j > m)$$

$$= P_j(A) + P_j(A') \leq \frac{1}{2}$$

$$\Pr[y_T = j | \mathcal{I}_j] \leq \frac{1}{2}$$

$\mathcal{I}^* \supseteq \mathcal{I}_j$

$k \geq 24$ $\mathcal{I}^* \subseteq \mathcal{I}$

$$\mathcal{I} \subseteq \mathcal{I}^* \supseteq \mathcal{I}_j \cup \mathcal{I}_{-j} \subset \mathcal{I}^* \quad \text{and} \quad \mathcal{I}^* \subseteq \mathcal{I}$$

$$\mathcal{I}_a^t = \{0, 1\}^t$$

$$\Omega = \prod_a \Omega_a^T \quad \text{and} \quad \mathcal{I}^* \subseteq \mathcal{I}$$

$$m = 24T/k - 1 \quad j \in K_3 \quad \mathcal{I}^* \subseteq \mathcal{I}$$

$$\Omega^* = \Omega_j^m \times \prod_{a \neq j} \Omega_a^T$$

$\mathcal{I}^* \supseteq \mathcal{I}_j$ $\mathcal{I}_j \in \mathcal{I}$

$$\forall A \subseteq \Omega^* \quad P_e^*(A) = \Pr[A | \mathcal{I}_e]$$

$P_g^* \rightarrow P_o^*$ $\mathcal{I}^* \supseteq \mathcal{I}_j$ $\mathcal{I}_j \in \mathcal{I}$

$$d(P_o^*(A) - P_j^*(A))^2 \leq KL(P_o^* || P_j^*) \quad (\text{ Pinsker })$$

$$= \sum_a \sum_{t=1}^T KL(P_o^{a,t} || P_j^{a,t}) \quad (\text{ chain })$$

$$= \underbrace{\sum_{a \neq j} \sum_{t=1}^T KL(P_o^{a,t} || P_j^{a,t})}_{0} + \sum_{t=1}^m KL(P_o^{j,t} || P_j^{j,t}) \leq 2\varepsilon^2$$

$$\leq 2\varepsilon^2 m$$

$$\varepsilon = \sqrt{\frac{ck}{T}} \quad \text{with } \gamma$$

$$E[\text{regret}] \geq \frac{T}{24} \sqrt{\frac{c_k}{T}} = \Omega(\sqrt{T k})$$

? What size is enough to do

מִתְבָּרְכָה מִתְבָּרְכָה מִתְבָּרְכָה

Asymptotic SF: Caveat

$$T \leq \frac{ck}{\varepsilon^2} \quad \text{and} \quad \text{with } \textcolor{red}{G(G)}.$$

לטנקי ארגון גוף נסיעות

$$\frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12} \leq P_{\alpha}[a_f \neq a^*] \quad \text{rho' run}$$

: 30 ଦିନ $a_t \neq a^*$ ହେଲୁ

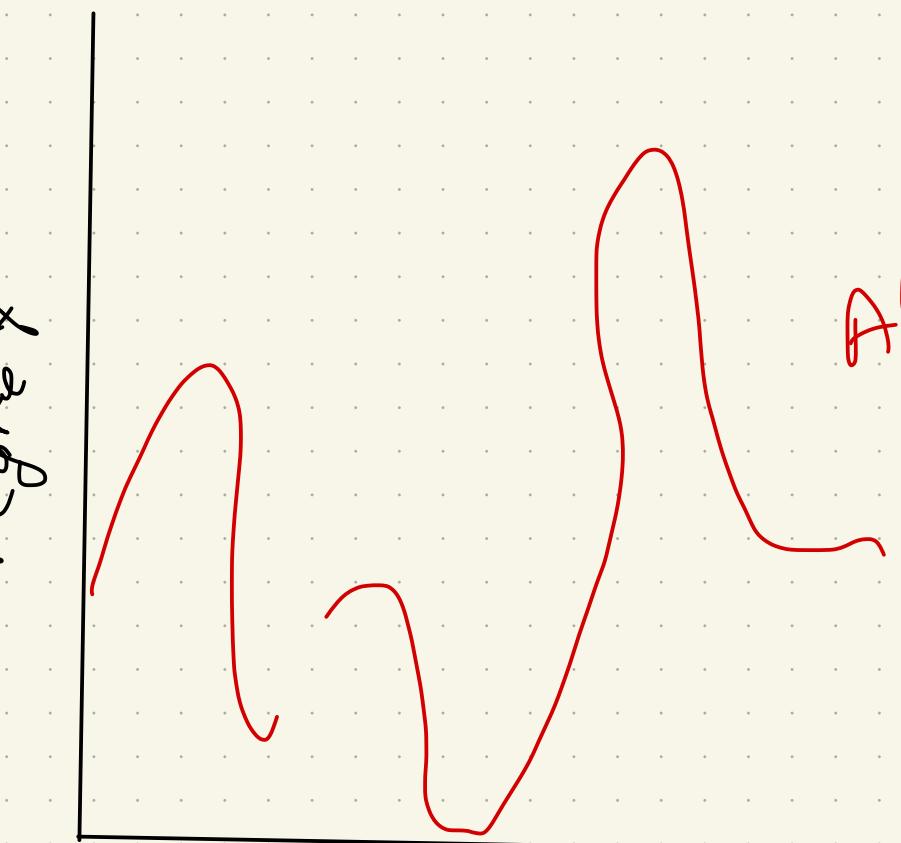
$$\Delta(\alpha_t) = \mu(\alpha^*) - \mu(\alpha_t) = \frac{1+\varepsilon}{2} - \frac{1}{2} = \frac{\varepsilon}{2}$$

$$E[\Delta(a_t)] = \Pr_r[a_t \neq a^*] \cdot \frac{\epsilon}{2} \geq \frac{\epsilon}{24}$$

$$E[\text{regret}] = \sum_{t=1}^T E[\Delta(a_t)] \geq \frac{T\varepsilon}{24}$$

Instance Dependent Lower Bounds

האם קיימת גבול תחתון
לפונקציית האלגוריתם?



אלגוריתם

בכל פעם מוציא פול

כלvc:

נקו גורן, ניקי, גרי, ניר
כלvc

טובי אלגוריתם

$$c_1 T^{0.1} \text{ כוונת}$$

שאלה: מתי מוגדרת כאלגוריתם?

$$T \geq 10^6 \text{ איט}$$

: כוונת

$$\mathbb{E}[\text{regret}] = \Omega\left(\frac{k}{\Delta} \log T\right)$$

$k=2$ מוגדר אלגוריתם

$$n_2 < \frac{0.9 \log T}{KL(\mu_2 || \lambda)} \quad : \text{E} \times \text{N} \text{ ג'זע}$$

$$E_p[n_2] \geq \frac{2}{3} \cdot \frac{0.9 \log T}{KL(\mu_2 || \lambda)}$$

$$= \sum \left(\frac{\log T}{KL(\mu_2 || \mu_1)} \right)$$

$$E_p[\text{regret}] = E_p[n_2](\mu_1 - \mu_2)$$

$$= \sum \left(\frac{(\mu_1 - \mu_2) \log T}{KL(\mu_2 || \mu_1)} \right) = \sum \left(\frac{\log T}{\Delta} \right)$$

$$P_p(E) < \gamma_3 \quad (K)$$

$$\mu_2 > N$$

$$(P) \quad \mu_1 > \mu_2 \quad (\mu_1, \mu_2) \quad \text{לפניהם}$$

$$(Q) \quad \mu_1 < \lambda \quad (\mu_1, \lambda) \quad \text{לפניהם}$$

$$KL(\mu_2 || \lambda) \approx 1.1 KL(\mu_2 || \mu_1)$$

1. ל'ג נבנה גורם $P \rightarrow$
2. ל'ג $Q \rightarrow$

מבחן 2 נסחף מינימום $-n_2$

$$E_Q[n_1] = E_Q[T - n_2] \leq c_1 T^{0.1}$$

$Q \rightarrow$ מבחן מוקדם

$$Pr_Q[n_2 < \frac{0.9 \log T}{KL(\mu_2 || \lambda)}] \leq \frac{E_Q[T - n_2]}{\frac{T - 0.9 \log T}{KL(\mu_2 || \lambda)}} \quad (\text{מרקורי})$$

$$\leq c_3 T^{-0.9}$$

: regret \geq for soon

$$E_p[\text{regret}] = E_p[n_2](\mu_1 - \mu_2)$$

$$= \Omega\left(\frac{\log T}{KL(\mu_2 || \mu_1)} (\mu_1 - \mu_2)\right)$$

$$= \Omega\left(\frac{\log T}{\Delta}\right)$$

means suffice

(0.6, 0.4)

ifc (0.4, 0.6) for each other

? REGRET $\geq N$

$O(1)$

ifc

$O(\log T)$

$$Pr_p(E) \geq \nu_3 \quad \textcircled{2}$$
$$P_{\alpha Q}[E] \leq c_3 T^{-0.9} \leq \nu_3 \quad \text{ifc} \Rightarrow$$

$$\begin{aligned} KL(P || Q) &\geq p(E) \log \frac{p(E)}{Q(E)} \\ &\geq \frac{1}{3} \log \frac{1/\nu_3}{Q(E)} \\ &\geq \frac{1}{3} \log \left(\frac{1}{3 \cdot c_3} T^{0.9} \right) = \Omega(\log T) \end{aligned}$$

$E_p[n_2] \propto O(T)$

$$KL(P || Q) = E_p[n_2] KL(\mu_2 || \lambda)$$

$$E_p[n_2] = \Omega\left(\frac{\log T}{KL(\mu_2 || \lambda)}\right)$$

$$= \Omega\left(\frac{\log T}{KL(\mu_2 || \mu_1)}\right)$$

: NLL/CE

2 چوں Slivkins

Instance Dependent

Kleinberg, Niculescu-Mizil, Sharma 2010

نک 3 چوں O(1)

وونا وونا وونا

. نک چوں - L_i

divergence - KL

: نک چوں وونا

نک چوں CN

MAB

: regret وونا

$\mathcal{O}(\sqrt{kT})$

worse case

$\mathcal{O}\left(\frac{\log T}{\Delta}\right)$

Instance based